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STOCHASTIC AND ADAPTIVE SYSTEMS.(U)

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STOCHASTIC AND ADAPTIVE SYSTEMS

(FINAL REPORT)

by

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Covering the Time Period (February 1, 1972 to January 31, 1977)

Submitted to: Charles L. Nefzger, Captain, USAF
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ABSTRACT

This final report describes the research carried out by members of the Decision and Control Sciences Group at the Electronic Systems Laboratory, M.I.T. during the time period February 1, 1972 to January 31, 1977 with support extended by the Air Force Office of Scientific Research under Grant AF-AFOSR 72-2273.

The principal investigators were Professor Michael Athans and Professor Sanjoy Mitter. The contract monitors were Lt. Colonel W.J. Rabe and Captain C.L. Nefzger of the AFOSR Directorate of Mathematical and Information Sciences.

Research was carried out on the following main topics:

1. Modelling and Identification of Stochastic Systems
2. Structure of Non-linear Stochastic Systems
3. Linear and Non-linear Filtering
4. Failure Detection Methods
5. Stochastic Control
6. Adaptive Control
7. Control and Estimation Problems for Systems with Time Delay

Technical details of the research may be found in the reports, theses, and papers cited in the references. A list of publications supported wholly or partially by this grant is included at the end of this report.

0. Introduction

In this report we describe the work on stochastic and adaptive systems which has been done during the period February 1, 1972 to January 31, 1977 supported by the grant AF-AFOSR 72-2273. The report is divided into two parts. In Part 1 of this report we describe the work that has been done during the period January 1, 1976 to January 31, 1977. A number of new investigations has been initiated during this period. In Part 2 of this report we give a summary of research that has been accomplished during the period February 1, 1972 to January 1, 1976. Details of this can be found in the list of publications which has been transmitted to Captain Nefzger earlier.

Research has been conducted on the following main topics:

Modelling and Identification of Stochastic Systems

Structure of Non-linear Stochastic Systems

Linear and Non-linear Filtering

Failure Detection Methods

Stochastic Control

Adaptive Control

Control and Estimation Problems for Systems with Time Delay

We remark that this work has applications to a wide class of aerospace problems.

0.1. Contributions Made in this Research Program

We feel that the major contributions of this work have been the following:

i) Modelling of $\frac{1}{f}$ noise and filtering in the presence of $\frac{1}{f}$ noise

ii) Performance Bounds for Non-linear Filters using Information Theory ideas and development of an informational framework for the study of non-linear filtering

iii) Explicit construction of finite dimensional optimal non-linear filters for a class of non-linear systems which are important in aerospace applications

iv) Important insights into Consistent Estimation and Dynamic System Identification

v) Stability Criteria for Feedback Systems with coloured noise gains

vi) Filtering theory for systems with time delay, both when the delay is known and when it is unknown and has to be estimated

vii) Solution of the one-step delay Linear Quadratic Gaussian problem

viii) Development of a theory and algorithm for estimation and control of finite state probabilistic systems over an infinite horizon using finite memory estimators and controllers

ix) Insights into steady-state optimum stochastic control of linear systems with random parameters

x) Development of a theory of estimation and control for a large class of infinite-dimensional linear systems

PART 1

Description of Research done during the Period

January 1, 1976 to January 31, 1977.

1. Introduction

Research has been conducted on the following topics:

i) Modelling and Identification of Stochastic Systems

ii) Nonlinear Estimation

iii) Stochastic Stability

iv) Geometric Theory of Control of Linear Stochastic Systems

v) Robust Linear Quadratic Design

vi) Linear Quadratic Gaussian Stochastic Control Problem

vii) Adaptive Control

viii) Control of Finite-State Stochastic Systems

ix) Finite-State Compensation.

This work is summarized in this section of the report. Further details can be found in the publications and theses referred to in the text.

2. Description of Research

2.1. Modelling and Identification

In order to do filtering and control using state-space methods it is necessary to have a Markov model of the stochastic system. Even if a Markov model were available the parameters of the system may be unknown and may have to be identified using noisy measurements. We have begun investigating these fundamental aspects of modelling of stochastic systems.

2.1.1. Stochastic Realization Theory

For finite-dimensional constant linear systems a sophisticated realization theory is now available. For stochastic linear systems very little seems to be available. In joint work with G. Picci [1] a theory of stochastic realizations has been initiated. The goal of this research is to construct a theory analogous to the theory of Kalman [2] for deterministic linear systems which would enable us to give an intrinsic construction of the innovations representation of second order stochastic processes. This work is being done by Professor Mitter. Further clarification of the results obtained in [1] have been made and will be reported in a forthcoming publication.

2.1.2. Information, Consistent Estimation, and Dynamic System Identification

Mr. Yoram Baram and Professor N.R. Sandell, Jr. have recently completed a study of the asymptotic behavior of various system identification methods. The study was motivated by another research project in ESL in which the dynamic equations of an aircraft were identified during flight for purposes of adaptive control. Due to computer limitations, the approach was to identify an appropriate model from a small finite set of models corresponding to various flight conditions of the aircraft. For this application, it was particularly obvious that the true system (even neglecting nonlinearities) would almost never be a member of the model set. Thus the question of the behavior of various identification techniques when the true model is not an admissible estimate arises naturally for this application. A related question is how to choose a finite number of models to best represent the true system. Note that both of these questions are quite distinct from the usual question of consistency of identification methods; the problem of consistency explicitly assumes that the true system is an admissible estimate.

With this background as motivation, the asymptotic behavior of parameter estimates and the identification and modeling of dynamical systems were investigated. Measures of the relevant information in a given sequence of observations were defined and shown to possess useful properties, such as the metric property on the parameter set. The convergence of maximum likelihood and related Bayesian estimates for general observation sequences were investigated. The situation where the true parameter is not a member of a given parameter set was considered as well as the situation where the parameter set includes the true model. The finite parameter set case was emphasized for simplicity in the convergence analysis, but the results were extended in general terms to the infinite parameter case. It was shown that under uniqueness conditions on the output, statistics of linear dynamical systems identification procedures converge to the true model if it is a member of the given model set. If the true model is not a member of the set, then the estimates converge to a model in the set, closest to the actual system in the information metric sense. Stationary and non-stationary systems were considered. Rates of convergence in the mean were obtained, and the separate contributions of the stochastic and the deterministic parts of the input to the convergence rates were shown. The analysis also suggested methods for approximating a high order system by a low order model and for selecting a representative model from a given model set, applicable to infinite and even non-compact model sets.

These results have been reported in the Ph.D. thesis of Mr. Baram [3].

References

1. G. Picci, "Stochastic Realization for Gaussian Processes", Proceedings of the IEEE, Vol. 64, No. 1, January 1976.

2. R.E. Kalman, P.L. Falb and M.A. Arbib, Topics in Mathematical System Theory, McGraw Hill, New York, 1968.

3. Y. Baram, "Information, Consistent Estimation and Dynamic System Identification", Massachusetts Institute of Technology, Cambridge, Massachusetts, Ph.D. Thesis, ESL-R-718, November 1976.

2.2. Non-linear Estimation

2.2.1. Design of Suboptimal and Efficient Optimal Estimators Using Spherical Harmonics and Fourier Analysis

During the past year we have continued our efforts to develop efficient and accurate estimation schemes for nonlinear systems. The results of this work are described in detail in [1,2] and further successful experimental results will be reported in [3]. Specifically work has been concentrated in two directions. In the first of these [1,3] we have applied techniques from the theory of spherical harmonics in order to obtain simple nonlinear filter design methodologies for certain systems. Recently, this methodology was applied to the design of a system for tracking the motion of a point on the surface of the sphere given noisy measurements of position. In all cases tested our filter outperformed or at least matched the performance of the extended Kalman filter and a linear minimum variance filter. In addition, our filter is far simpler to implement. Given these promising results, we are considering extending these ideas to a detailed look at satellite tracking and inertial navigation problems.

The second direction, as discussed in [2], consists of the examination of estimation for certain finite state systems. As many nonlinear filtering systems are to be implemented in a discrete fashion--i.e. on a digital computer--one can think of taking continuous-state systems and approximating them by discrete-state models. Of course, one will in general obtain models with extremely large numbers of states, and the combinatorics of the optimal estimation problem are imposing. It is precisely this issue that we address. Specifically, with the aid of the tools of finite Fourier analysis, we have developed extremely efficient algorithms for estimation in certain special cases. Future work will involve loosening the restrictions under which these techniques can be applied.

Estimation for Systems with Unknown Time Delays

The problem considered in [4] is one of practical importance in a variety of applications. A transmitter sends out a signal which is sensed by one or more sensors, but the transmission time from the transmitter to each sensor is only described in a probabilistic fashion. In addition, the signal may be reflected, leading to multipath observations. We would like to estimate both the transmitted signal and the transmission time delays. In [4] we have considered several simplified versions of this problem, have obtained implementable optimal estimation equations in certain cases, and have uncovered some of the key issues involved in the design of recursive systems for such problems. Work will continue in the development of usable algorithms for problems of this type.

References

1. S.I. Marcus and A.S. Willsky, "The Use of Spherical Harmonics in Suboptimal Estimator Design", Proceedings 1976 IEEE Conference on Decision and Control, Clearwater Beach, Florida, December 1976.

2. A.S. Willsky, "The Use of Group-Theoretic Concepts in Solving Problems in Estimation and Control", Proceedings 1976 IEEE Conference on Decision and Control, Clearwater Beach, Florida, December 1976.

3. S.I. Marcus and A.S. Willsky, "Suboptimal Estimator Design Using Spherical Harmonics: Analysis and Numerical Results", to be submitted to IEEE Trans. on Automatic Control.

4. P.Y. Kam and A.S. Willsky, "Some Modeling and Estimation Problems Involving Space-Time Stochastic Systems", Proceedings 1976 IEEE Conference on Decision and Control, Clearwater Beach, Florida, December 1976.

2.2.2. Stable Non-linear Observers

Mr. Safonov and Professor Athans have completed work which discusses a new type of nonlinear estimator design, designated the Suboptimal Nonlinear Observer (SNO). The SNO constitutes a simple, practical alternative to the Extended Kalman Filter for nonlinear estimation.

A hybrid between the Kalman Filter and the Nonlinear Observer*, the SNO consists of a nonlinear internal reference model of the system dynamics with observations entering via a memoryless gain acting on the residual error between predicted and actual observations. The distinguishing feature of the SNO is the way in which its residual gain is chosen: it is chosen to be the Kalman filter gain for a nominal time-invariant linearization of the system. This choice leads to an estimator design which maintains to a first approximation the minimum-variance optimality of the Extended Kalman Filter but is much simpler in structure.

The results obtained concerning SNO estimators include analytically verifiable conditions for a SNO design to be assured of being nondivergent. These conditions are constructive in the sense that they provide the engineer with information that is helpful in making design modifications necessary to correct divergence problems.

The SNO estimator design procedure has been found to be robust in the sense that the class of nonlinear systems for which SNO designs are assured of being nondivergent is large; this tolerance of nonlinearity is mathematically

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*T.-J. Tarn and Y. Rasis, "Observers for Nonlinear Stochastic Systems", IEEE Trans. on Automatic Control, Vol. AC-21, August 1976, pp. 441-448.

dual to the previously reported robustness of linear-quadratic state-feedback systems against variations in open-loop dynamics and unmodelled nonlinearity.

It has been proved that substitution of the estimates generated by a nondivergent SNO for the true values in an otherwise stable feedback control system can never destabilize the closed-loop system. This result has important practical implications regarding the utility of SNO estimators for state reconstruction in nonlinear optimal and suboptimal feedback control systems.

2.3. Stochastic Stability

Following the work of Willems and Blankenship [1] on the stochastic stability of linear systems with a white-noise multiplicative feedback gain, Martin [2] studied the more realistic case of a scalar colored-noise multiplicative feedback gain. This problem is closely-related to the problem of assessing closed-loop performance due to a stochastic control law, where the control law itself may be random if it depends on past observations of system outputs or on parameter estimates. The case of scalar colored-noise gains studied by Martin (see also Martin and Johnson [3,4]) requires substantially more than a mere formal extension of the earlier white-noise results. The algebraic complexity of the moment expressions and their bounds previously prevented extension of Martin's results to the multivariable case; during 1976 this hurdle was overcome and results will be presented in a future paper in the IEEE Transactions on Automatic Control, supported by this grant.

One of the most remarkable features of such systems, implicit in early work of Brockett and willems [5], is that moment stability does not imply almost-sure stability; in fact, the conditions for stability of the higher-order moments become progressively more severe (see [3],[4]). Related problems have been studied by Marcus [6]. There are reasonable parameter values for which almost every sample path is uniformly bounded (in the initial state) but all moments become unbounded. The significance of this curiosity did not become apparent until the recent investigations of Ku and Athans [7], reported elsewhere herein. For good reasons, the use of mean-square estimation and control criteria has been common in both linear and nonlinear stochastic control problems. Approximate solutions of such problems have often presumed moment stability. Particularly in cases where uncertain or slowly-varying parameters enter multiplicatively, mean-square criteria may yield closed-loop stochastic stability requirements which are too conservative. These considerations may require a fundamental reassessment of our formulation of stochastic control problems; continuing research is under way.

References

1. J.C. Willems and G. Blankenship, "Frequency Domain Stability Criteria for Stochastic Systems", IEEE Trans. Auto. Contr., Vol. AC-16, August 1971.

2. D.N. Martin, "Stability Criteria for Systems with Colored Multiplicative Noise", ESL Report 567, M.I.T., (September, 1974).

3. D.N. Martin and T.L. Johnson, "Stability Criteria for Discrete-Time Systems with Colored Multiplicative Noise", Proc. 1975 Conference on Decision and Control, IEEE, (Houston, Tex.).

4. D.N. Martin and T.L. Johnson, "Stability Criteria for Continuous-Time Systems with Colored Multiplicative Noise", p. 1108-1113, Proc. 1976 Conference on Decision and Control, IEEE (Clearwater, Florida).

5. R.W. Brockett and J.C. Willems, "Average Value Criteria for Stochastic Stability", Proceedings of the Symposium on Stochastic Dynamical Systems, Math. Lecture Notes No. 294, Springer-Verlag, New York, 1972.

6. S.I. Marcus, "Estimation and Analysis of Nonlinear Stochastic Systems", Ph.D. Thesis, Dept. of Elect. Eng., M.I.T., Cambridge, Mass., June, 1975.

7. M. Athans, R. Ku, and S.B. Gershwin, "The Uncertainty Threshold Principle: Fundamental Limitations of Optimal Decision Making under Dynamic Uncertainty", Proc. 1976 Conf. Dec. & Cont., p. 1142-1145, IEEE (Clearwater, Florida).

2.4. Geometric Theory of Control of Linear Stochastic Systems

Professor Mitter has begun development of qualitative theory of control of linear stochastic systems analogous to the geometric theory of control of linear deterministic systems as developed by Wonham [1]. The first task in such a development is the need to define the concept of a stochastically unobservable subspace. This in turn necessitates a detailed study of the Riccati equation without any assumptions of controllability and observability. An examination of the literature on the Riccati equation has shown that very little is available in this direction and what is available seems to be incorrect. No results have been obtained so far but an understanding of the nature of results to be obtained has been gained.

Reference

1. W.M. Wonham, Linear Multivariable Control, *Springer Lecture Notes*, 1975.

2.5. Closed-Loop Stability Robustness of Multiloop LQ-Optimal Design

New results have been obtained characterizing the robustness of the closed-loop system stability property of multiloop LQ-design is the presence of perturbations in the feedback gains as well as in the open-loop system parameters. These new results unify and generalize known stability margin properties of LQ-design. The new robustness measures are expressed explicitly in terms of the weighting matrices in the quadratic cost index. These explicit relationships provide a new basis for studying the effects of the choice of cost criteria on the closed-loop stability robustness of the corresponding "optimal"-design. In particular, the questions of system reliability (feedback integrity) to failures in the feedback loops can be investigated using the new results we obtained.

The results of our research are reported in:

P.K. Wong, M. Athans, "Closed-Loop Stability Robustness of Multiloop Linear Quadratic Optimal Design." (forthcoming).

2.6. Linear-Quadratic Gaussian Stochastic Control Problems

2.6.1. There are countless formulations of LQG stochastic control problems, and a great number of these have been analyzed in periodicals; yet, surprises of fundamental import may be found here. In a series of papers we have studied control law simplifications which result when the stochastic control problem is nearly singular, in the sense that plant process or observation noises may be negligably small. In these cases, the standard Kalman filter/quadratic regulator solution will be unsatisfactory in steady state; when process noise is absent, the filter gains go to zero (the filter is oblivious to new information), and in the absence of observation noise, they become infinite (because they depend on the inverse of the covariance). In Platzman and Johnson it has been shown that an output-feedthrough term can be determined in such a way that the foregoing problems are avoided; this requires an innovations-decomposition of the control law and an extension of the LQG performance index to penalize the innovations part of the control. The extended problem does not give rise to the anomalies of the usual LQG problem, which are often an enigma in practical computation of optimal regulators; it may provide a general setting which ensures uniform convergence of stochastic controls in the case of small plant or observation noise. In the case where some observations are noiseless, Rom and Sarachik demonstrated that the Bryson-Johanson filter followed by the Kalman gains is an optimum solution of the singular LQG problem, i.e. a reduced-order filter (equivalent to a minimal-order observer) can be used. In [2], we have studied the algebraic structure of this problem and showed that, in principle, it is possible to determine the optimum compensator's transfer function directly and uniquely from the plant transfer function and the noise and performance index parameters. In [3], we have shown that the solution of a dual problem, where the process noise is singular is a dual-observer based compensator whose optimum gains are in complete algebraic duality to the optimum gains of Rom and Sarachik. These results provide insight for the design of low-order approximate stochastic control in the case where some of the plant and/or observation disturbances are very small.

References

1. L.K. Platzman and T.L. Johnson, "A Linear-Quadratic-Gaussian Control Problem with Innovations-Feedthrough Solution", IEEE Trans. Auto. Control, Vol. 21, No. 5.

2. P. Blanvillain and T.L. Johnson, "Invariants of Optimal Minimal-Order Observer-Based Compensators", Proc. 1976 IEEE Conference on Decision and Control, pp. 186-190, (Clearwater, Florida), Reference No. 10.

3. P.J. Blanvillain and T.L. Johnson, "Specific-Optimal Control with a Dual Minimal-Order Observer-Based Compensator", submitted to Int. J. Control, August, 1976.

4. T.L. Johnson, "Finite-State Compensators for Physical Systems", ESL Technical Memorandum No. 658, April, 1976.

2.6.2. Optimal Estimation and Stochastic Control for Interconnected Systems

Two of the major problems with the standard Kalman filter and linear-quadratic-Gaussian regulator are that it is quite difficult to solve the required Riccati equations in order to determine the optimal gains and also the required on-line computation for these designs may be exorbitant. Recently [Ref. 2, Section 2.2.1.] we have begun to investigate a class of stochastic control and estimation problems for which efficient solutions can be obtained with the aid of fast Fourier transform techniques. These systems consist of a collection of identical subsystems which are coupled together in a symmetric fashion. Our initial results for these systems are quite promising, and we plan to continue the study of such systems and the applicability of our techniques to systems which may not exactly satisfy the structural symmetry assumptions imposed in the reference cited above.

2.7. Adaptive Control of Systems with Purely Random Parameters

Professor Athans and Mr. Richard Ku have continued their work on Adaptive Control. Progress has been made on the problem of optimal control of stochastic linear systems with stochastic parameters. In particular, the system is assumed to be a linear plant with randomly varying matrices A and B perturbed by additive white Gaussian noise,

$$\underline{x}_{t+1} = A_t \underline{x}_t + B_t \underline{u}_t + \underline{\xi}_t \qquad t = 0,1,\ldots,T-1$$

It is assumed that the randomly varying parameters (A_t, B_t) and plant noise $\underline{\xi}_t$ are independent of each other for all t. Perfect measurements

$\underline{y}_t = \underline{x}_t$ are assumed.

The controls are restricted to the class of nonrandomized control strategies of the form

$$\underline{u}_t = \phi_t(\mathcal{Y}^t, \mathcal{U}^{t-1})$$

where $\mathcal{Y}^t = \{\underline{y}_0, \ldots, \underline{y}_t\}$ and $\mathcal{U}^{t-1} = \{\underline{u}_0, \ldots, \underline{u}_{t-1}\}$.

Given the system structure and information pattern above, it is desired to find the optimal control law $\{\underline{u}_t : t = 0,1,\ldots,T-1\}$ such that the expected value of the quadratic cost functional

$$J = \underline{x}_T' F \, \underline{x}_T + \sum_{s=0}^{T-1} \underline{x}_s' Q_s \underline{x}_s + \underline{u}_s' R_s \underline{u}_s$$

is minimized. We assume that $Q_s \geq 0$ and $R_s > 0$.

It is found that for the one-dimensional problem, that the optimal control law is of the form

$$u_t^* = -G_t \underline{x}_t$$

where the non-random control gain is given by

$$G_t = \frac{K_{t+1}(\Sigma_{ab}(t) + \bar{a}_t \bar{b}_t)}{R_t + K_{t+1}(\Sigma_{bb}(t) + \bar{b}_t^2)}$$

The scalars K_t are computed from the backward difference equation

$$K_t = Q_t + K_{t+1} (\Sigma_{aa}(t) + \bar{a}_t^2) - \frac{K_{t+1}^2(\Sigma_{ab}(t) + \bar{a}_t\bar{b}_t)^2}{R_t + K_{t+1}(\Sigma_{bb}(t) + \bar{b}_t^2)}$$

$$K(T) = F.$$

It is of interest to compute the optimal stationary control for a linear time-invariant system with stationary statistics. For the scalar system, it is found that the solution to the Riccati equation may not be finite.

If we define $m = \Sigma_{aa} + \bar{a}^2 - \dfrac{(\Sigma_{ab} + \bar{a}\bar{b})^2}{\Sigma_{bb} + \bar{b}^2}$,

For $m > 1$, the backwards solution K_t becomes unbounded. The expected optimal cost is therefore unbounded. Hence, the optimal control system is asymptotically unstable in the mean square sense. Since G_t has a constant value even for large K_t, the sample path x_t may be stable, but the mean-square sense stability of x_t cannot be established.

Necessary and sufficient conditions for a randomly varying system to be mean-square asymptotically stable is that the combination of levels of parameter uncertainties given by m be less than one. The theorem holds for the more complex multivariable system as well.

From this important result, various concepts of stochastic stability, stochastic controllability and observability may be established. Furthermore, the control law for the system with independent parameter variations clarifies the notion of dual control and the inherent caution and probing phenomenon in the interaction of control and identification.

2.8. Control of Finite State Stochastic Systems

2.8.1. Finite Memory Control and Estimation of Finite Probabilistic Systems

A finite probabilistic system (FPS) is a stationary discrete-time controlled stochastic dynamical process, having finite input, output, and (internal) state sets. The partially-observable Markov decision process is an example of such a system. FPS formulations provide a convenient framework for the study of problems of state estimation, statistical decision, or control, where state information is available only through a finite memoryless channel, and observation dynamics may depend on the inputs selected.

Notions of reachability and detectability in FPS's (similar to controllability and observability in linear systems) have been made precise. It has been shown that every FPS can be reduced to components that are either reachable and detectable, or transient, or null-recurrent.

It has been well known that the information vector (whose i-th entry is the *a posteriori* probability that the system is in state i) is a sufficient statistic (for the estimation of future dynamics given past inputs and outputs). A contraction property of the information vector transition function has been exploited to obtain procedures for ε-optimal (arbitrarily close) approximation of the information vector by a deterministic time-invariant finite-memory observer. Each observer state corresponded to a particular configuration of most recent input-output pairs. The average error achieved by such an approximation has been bounded by the expression $(m/m_0)^{-\tau}$, where m_0 and τ are parameters associated with the observed system, and m is the number of observer states.

Control problems, in which the average reward has been maximized over a discounted or undiscounted infinite horizon, may be solved by an iterative procedure which has been given the name *perceptive dynamic programming*. Successively weaker assumptions that the controller "perceives" unavailable state values has transformed the problem into a sequence of formulations which may be solved by dynamic programming. Each solution obtained in this manner has been used to construct a feasible controller formulation, taking the form of a deterministic time-invariant finite-state automation. Monotone geometrically convergent bounds, containing both the supremum feasible performance and that of the current design, have also been obtained. Computation may be terminated when these bounds have become sufficiently close, or when the number of controller states has become excessively large. Although computing a solution by perceptive dynamic programming may require considerable time and storage, both have been roughly proportional to the number of controller states allowed in the final iteration; thus the cost of controller design has reflected the cost of controller implementation.

This procedure was applied to idealized problems of machine maintenance and computer communication, both of which had been investigated by other researchers. The first problem was solved exactly; a design suitable cost to the optimum was obtained for the second problem.

Details of this work can be bound in the doctoral dissertation of L.K. Platzman entitled "Finite Memory Estimation and Control of Finite Probabilistic Systems" written under the direction of Professor Mitter.

2.8.2. Finite-State Compensation

Aside from linear-quadratic-Gaussian stochastic control problems, the only substantial class of processes for which stochastic controls might be considered "computable" are Markov chains. In this case policy improvement algorithms may converge in a finite number of steps (the number of possible control laws is denumerable, under appropriate assumptions). Furthermore, such control laws are directly implementable by finite-state sequential machines having sufficient memory. Problems for which time and/or states take on continuous values in general lead to very large-scale mathematical programming problems for the numerical approximation of the feedback operator, which is generally infinite-dimensional. The ideas presented in [cf. Ref. 4, Section 2.6.1.] relate to this problem. We have proposed that in cases where the physical variables of a continuous system may be apriori bounded, then it is in principle possible to <u>uniformly</u> approximate the optimum control law by a finite-state (possibly asynchronous) machine. A decomposition of the feedback mapping is also proposed. While the ideas put forward in [Ref. 5, Section 2.6.1.] are based on extensive practical experience, they must yet be made more precise, initially through consideration of some realistic examples.

PART 2

Description of Research done during the Period

February 1, 1972 to January 1, 1976

1. Summary Description of Research Conducted During the Period 1972-1976

1.1 Modelling of $\frac{1}{f}$ Noise

A study was undertaken to model gyroscopic noise which had been experimentally found to have a spectral density exhibiting $\frac{1}{f}$ behavior. The final objective was to do filtering in the presence of gyroscopic noise.

A linear infinite dimensional model of gyroscopic noise was constructed. The filtering problem in the presence of gyroscopic noise was solved completely. It was shown that under appropriate detectability and stabilizability hypotheses the filter was asymptotically stable. The filter is naturally infinite dimensional. The problem of finite dimensional approximation of the filter was investigated and upper and lower performance bounds of the approximate filter were obtained.

References

1. L.B. Horowitz, "Optimal Filtering of Gyroscopic Noise", Ph.D. Thesis, June 1974, Massachusetts Institute of Technology, Cambridge, Massachusetts.

2. L.B. Horowitz and S.K. Mitter, "An Example of an Infinite Dimensional Filtering Problem: Filtering for Gyroscopic Noise", Proceedings of the 1976 Conference on Decision and Control, IEEE Control Systems Society, 76CH1150-2CS, Clearwater, Florida, December 1976.

1.2 Non-linear Filtering

Research on various aspects of non-linear filtering was done by Professors Mitter and Willsky with aid of Drs. K.P. Dunn, J. Galdos and S. Marcus.

1.2.1 Performance Bounds for Non-linear Filtering using Information Theory Ideas

The importance of the nonlinear filtering problem in aerospace applications is well known. The problem arises where one has a physical situation

Space-Time Stochastic Systems", Proceedings 1976 IEEE Conference on Decision and Control, Clearwater Beach, Florida, December 1976.

modeled by nonlinear stochastic differential equations, and observations are made by nonlinear noisy sensors. The problem is particularly relevant when there is more than a "mild" nonlinearity so that the usual linearizing techniques fail to describe the situation accurately.

The state of affairs in nonlinear filtering can be briefly described as follows. Progress has been produced in two largely disjointed areas: theory and practice. The main results in the former area are the well known representation theorems [5] (Kushner-Stratonovich, Bucy-Mortensen-Duncan, Frost) characterizing the fundamental filtering entity, the conditional distribution. In practice, ad-hoc methods based on extensions of linear theory and various parameterization of the conditional density have been used to build feasible filters. These suffer from the following drawbacks: Even the techniques that are considered to be "better" diverge in simple examples; their performance cannot be apriori assessed except perhaps through brute force Monte Carlo simulations; Monte Carlo simulations are unreliable and are known in some cases to produce over-optimistic results.

The overall objective of this study has been to probe into nonlinear filtering by analytical methods rather than by experimental methods whose inadequacy has been considered in the previous paragraph. The specific analytical tool used has been Shannon's Information Theory.

Information theory was developed by Shannon [2] in the late 1940's to cast communication problems into a mathematical framework. Information theory has as its point of departure a precise definition of uncertainty in terms of information quantities (in particular mutual information). It then goes on to characterize the communication problem from a coding-decoding point of view and gives conditions under which reliable communication (acceptable distortion) can be achieved -- all in terms of the aforementioned information quantities. The bridge between information theory and filtering is therefore the relation between uncertainty as characterized by information quantities and uncertainty as characterized by mean square (MS) distortion in the filtering context.

The fundamental work in the information theory context relating information to distortion was done by Shannon [3] who formulated what is now known as Rate Distortion Theory. Work aimed at relating information theory to dynamical systems and filtering has been done by Gray [6], Toms [7], Goeblick [4], and Weidemann [1]. The common difficulty found in these studies is that they attempt to imbed dynamical systems and filtering into the classical information theory context which is asymptotic in nature and with interval performance indices. In contrast modern filtering is essentially a real time (causal) problem and the filtering performance index is a point performance index (MSE). In this study the filtering problem has been embedded into a modified information context which is far more natural and which allows the clearer establishment of the relation between information and MS distortion.

Potentially this formulation may allow the imposition of an information framework for the design and comparison of nonlinear filters.

At a more immediately applicable level this study has resulted in the development of a lower bound on filtering MSE based on information quantities (rate distortion theory). A number of such lower bounds have appeared in the literature but all of them suffer from the drawback that they lose their usefulness when considerable nonlinearity exists. Recently Zakai and Ziv [8] produced a bound based on rate distortion theory which in addition to the linearity consideration was not applicable to discrete time systems and thus not applicable, for example, to radar tracking. The bound described in this study is based on the Bucy representation [5] which in addition to being easily applicable to discrete systems, provides a tighter bound on the rate distortion function and hence on MSE.

In terms of usefulness, a lower bound on MSE is an excellent design tool both before and during the filter design -- allowing the engineer to assess whether the design is feasible for the particular sensor under consideration as well as providing an absolute with which to compare the different possible designs. Details of our work can be found in [9].

References

1. H.L. Weidemann, E.B. Stear, "Entropy Analysis of Estimating Systems", IEEE Trans. Info. Th., Vol. IT-16, No. 3, May 1970.

2. C.E. Shannon and W. Weaver, "The Mathematical Theory of Communication", University of Illinois Press, 1949.

3. C.E. Shannon, "Coding Theorems for a Discrete Source with a Fidelity Criterion", IRE Natl. Com. Rec., Part 4, 1959.

4. T.J. Goblick, "Theoretical Limitations on the Transmission of Data from Analogue Sources", IEEE Trans. Info. Th., Vol. IT-11, No. 4, October 1965.

5. R.S. Bucy and P.D. Joseph, "Filtering for Stochastic Processes with Applications to Guidance", John Wiley, 1968.

6. R. Gray, "Information Rates of Autoregressive Sources," Ph.D. Thesis, University of Southern California, 1969.

7. W.E. Toms, "Information Rates of Dynamic Systems", Ph.D. Thesis, Cornell University, 1971.

8. M. Zakai, J. Ziv, "Lower and Upper Bounds on Optimal Filtering Error of Certain Diffusion Processes," IEEE Trans. Info. Th., Vol. IT-18, No. 3, May 1972.

9. J. Galdos, "Information and Distortion in Filtering Theory", Ph.D. Dissertation, January 1975, Massachusetts Institute of Technology, Cambridge, Massachusetts.

1.2.2. Cone-Bounded Nonlinearities and Estimation Bounds

It is well-known that in nonlinear estimation problems the determination of the true optimum estimator or its performance requires a computationally-intractable infinite-dimensional on-line calculation. This has motivated work on the determination of sub-optimal estimators and the study of their performance. But the exact evaluation of the performance of a sub-optimal estimator is in general not an easy problem. Recently, Rhodes and Gilman [1], [2] have established upper and lower performance bounds on filtering problems associated with a special class of nonlinear systems, which are computationally-tractable, rigorously-derived and asymptotically-tight. The main objective of the current research is to extend these results to smoothing and prediction problems associated with the same class of nonlinear systems, and also to include the situation when system and measurement noises are correlated.

The particular systems studied are systems with cone-bounded nonlinearities in the sense that, when modeled by Itō differential equations, they contain drift coefficients which are, to within a uniformly Lipschitz residual, jointly linear in the system state and externally applied control. The estimation bounds obtained in this research are upper bounds on the performance of some simple, almost linear predictor and lower bounds on the estimation error covariance attainable by any smoother or any predictor, including the optimum. They are solely determined by the cone-bounded nature of the nonlinearities and can be easily computed. Furthermore, they are independent of the control or control law. All bounds have the property that they converge uniformly to the optimum performance of the nominal linear problem as the nonlinearities tend to zero.

References

1. A.S. Gilman and I.B. Rhodes, "Cone-Bounded Nonlinearities and Mean-Square Bounds--Estimation Upper Bound", IEEE Trans. on Automatic Control, Vol. AC-18, No. 3, 1973, pp. 260-265.

2. I.B. Rhodes and A.S. Filman, "Cone-Bounded Nonlinearities and Mean-Square Bounds-Estimation Lower Bound", to appear, IEEE Trans. on Automatic Control.

3. K.P. Dunn and I.B. Rhodes, "Cone-Bounded Nonlinearities and Mean-Square Bounds-Smoothing and Prediction Bounds", to be presented at IFAC Conference of 1975 at Boston.

1.2.3. Analysis and Estimation of Nonlinear Stochastic Systems Possessing Certain Types of Structures

Professor Willsky with the aid of Dr. Marcus and other graduate students has done work in the area of stochastic analysis for nonlinear systems possessing certain types of structure. The major results obtained in this area arose out of the consideration of the following bilinear estimation problem

$$\dot{x}(t) = [A_0 + \sum_{i=1}^{N} A_i \xi_i(t)]x(t) \tag{1}$$

$$\dot{\xi}(t) = F\xi(t) + \dot{w}(t) \tag{2}$$

where we observe

$$z_1(t) = H\xi(t) + v_1(t) \tag{3}$$

$$z_2(t) = Hx(t) + v_2(t) \tag{4}$$

The case in which we observe the process z_1 is motivated by strapdown navigation problems, in which ξ can be interpreted as an angular velocity vector. This class of problems has yielded one of the first large class of nonlinear filtering problems for which there exist explicit, finite-dimensional optimal solutions. The basic method behind the work reported in [10-12] is the expansion of the solution of (1) in a <u>Volterra series</u>

$$x(t) = g_0(t) + \sum_{i=1}^{N} \int_0^t \mu_1^{\,i}(s)\xi_i(s)\,dx$$

$$+ \sum_{i,j=1}^{N} \int_0^t \int_0^\tau \mu_2^{ij}(\tau,s)\xi_i(s)\xi_j(t)\,ds\,dt + \ldots \qquad (5)$$

One of the major results obtained is a set of finite-dimensional nonlinear stochastic equations for the optimal estimate of each term in this series. Thus if the series is finite -- which is equivalent to an algebraic condition on the A_i in (1) -- one obtains a finite-dimensional optimal estimate for x. Generalizations of this to some cases in which (5) need not be finite are given in [1-3]. In addition, in these references it is shown that <u>many</u> nonlinear systems have representations as Volterra series as in (5), and optimal estimation schemes for these are derived.

In the cases in which the series (5) does not lead to finite-dimensional optimal estimators, one can use an approximation technique to truncate the series. The utility of this -- especially for the strapdown problem -- is being investigated.

If one wants to consider estimation given the measurement (4), the problem is much more difficult. These models arise in synchronous communication, satellite tracking, and incorporation of measurements such as from a star tracker into an estimate of attitude. The problem (1), (2), (4) leads immediately to infinite-dimensional filter equations. For the phase tracking problem, Willsky had earlier used a Fourier series method to obtain suboptimal estimators. Work has been done to obtain a priori performance bounds for the phase tracking systems and to develop alternative designs. In addition, Marcus [1] has extended these concepts to the design of harmonic-analysis-based nonlinear filters for satellite tracking and attitude estimation.

References

1. S.I. Marcus, "Estimation and Analysis of Nonlinear Stochastic Systems", Ph.D. Dissertation, also Report ESL-R-601, June 1975.
2. S.I. Marcus and A.S. Willsky, "Algebraic Structure and Finite Dimensional Nonlinear Estimation", Proc. Symposium Algebraic System Theory, Udine, Italy, June 1975; extended version submitted to SIAM J. Control.
3. A.S. Willsky and S.I. Marcus, "Estimation for Certain Nonlinear Stochastic Systems," Seminaires IRIA, Rocquencourt, France, May 1975.

1.3 Filtering for Delay Differential Systems

Work on filtering for linear and nonlinear time delay systems has been done. This work has been done by Dr. R.W. Kwong, Prof. A.S. Willsky, and Prof. S.K. Mitter. Briefly, the problem under consideration is described by the functional differential equation

$$dx(t) = f(\tilde{x}_t)dt + dw(t) \tag{1}$$

where $\tilde{x}_t$ is the state of the system defined by

$$\tilde{x}_t(\theta) = x(t+\theta) \qquad -\tau \leq \tau \leq 0 \tag{2}$$

Thus, (1) includes some time delay effects. Similarly, the measurement equation is

$$dz(t) = h(\tilde{x}_t)dt + dv(t) \tag{3}$$

and in this model one can consider "multipath" or "echo" effects -- e.g.

$$h(\tilde{x}_t) = ax(t) + bx(t-\tau) \tag{4}$$

The major results obtained are as follows. A general integral representation formula has been obtained for the conditional estimate

$$E[\phi(\tilde{x}_t)|z(s),\ 0 \leq s \leq t] \tag{5}$$

of any function $\phi(\tilde{x}_t)$ of the trajectory piece $\tilde{x}_t$. Recursive filtering equations are shown to exist in the linear case and in the nonlinear case when h and f satisfy certain smoothness conditions (e.g., pure point delays cannot be accomodated and one needs some type of "blurring")

$$h(\tilde{x}_t) = \int_{-\tau}^{0} p(\theta)x(t + \theta)d\theta.$$

In the linear-point delay case, however, a full set of filter equations are rigorously derived, and, in addition, the stability of the steady-state filter is shown when the only delays are in the dynamics (1) and when certain observability-type conditions are satisfied.

These results have been extended to the extremely important problem when the time delay τ is not known. Examples of this arise in various signal processing problems in which one is trying to estimate the distance from transmitter to receiver (or reflector) given that multipath effects may corrupt the received signal.

Using methods to study the infinite-time quadratic cost problem for infinite dimensional systems Professor Mitter (in collaboration with Dr. R.B. Vinter, formerly at M.I.T. and now at Imperial College) has been able to prove the stability of the filter (in the linear case) using hypotheses of stabilizability and detectability. The stabilizability and detectability conditions can be verified using matrix tests. The filter obtained is however infinite dimensional and work is now in progress to understand how the filter can be implemented using adders, multiplier and delay elements.

References

1. R.W. Kwong, Structural Properties and Estimation of Delay Systems, Ph.D. Thesis, Report ESL-R-614, Electronic Systems Laboratory, Massachusetts Institute of Technology, Cambridge, Mass., September 1975.

2. S.K. Mitter and R.B. Vinter, "Filtering for Linear Stochastic Hereditary Differential Systems, in Control Theory, Numerical Methods and Computer Systems Modelling", edited by A. Bensoussan and J.L. Lions, Springer-Verlag, New York, pp. 1-21, 1975 (also, presented at the International Symposium on Control Theory, Numerical Methods and Computer System Modeling, Institut de Recherche D'Informatique et D'Automatique, France, June 1974.)

1.4 The Stability of Linear Systems with Non-White Multiplicative Noise

Often, the mathematical model of a dynamical system requires the consideration of uncertain elements. Historically, emphasis has been placed on studying systems in which disturbances enter additively, but many control problems include disturbances that can be more naturally modeled as uncertain multiplicative gains. If the multiplicative gain is of the white noise type, necessary and sufficient conditions for the stability of the system have been derived. These results have been extended where the multiplicative gain is of correlated noise type.

This research has been done by Dr. David Martin in conjunction with Professors Johnson and Willsky. Systems with multiplicative correlated noise in the gain elements of the feedback path include oscillators, certain phase-locked loop configurations, and recursive computational algorithms, as well as human operator-controlled systems. Hence, stability criteria for such systems may be of considerable interest in Air Force Applications. The use of colored-noise gain elements is both more realistic and technically much more difficult than the previously-treated case of white-noise gains. Two theoretically results of fundamental importance have been established. In the continuous-time case, a rigorous proof of existence of solutions to the stochastic differential equations characterizing such systems has been developed. In the discrete-time case, the Markov property of the solution sequence has been proved. Both of these theorems establish the class of functions to which solutions of the equations must belong, which is a necessary prerequisite for proper stability results.

For the special case of continuous-time first-order systems with first-order colored noise gain processes, explicit stability criteria are available. The effect of correlation time and average gain can be evaluated both for "almost sure" and moment stability. Certain values of gain and correlation time can be found which yield "stable" sample paths but for which the system is unstable in the mean-square sense. An approximate analog simulation of this situation was carried out in order to gain a qualitative understanding of the behavior of such sample paths. To the extent that effects such as nonideal noise generation, amplifier saturation, and multiplier imperfections can be ignored, it appears that the sample trajectories in this case exhibit "burst" disturbances, i.e., periods of quiesence interspersed with episodes of large drift or oscillatory disturbances. In this qualitative sense, the simulation is in agreement with published studies of long-term stability of commercial electronic oscillators.

For first-order continuous-time systems with higher-order noisy gain processes bounds on stability regions in parameter space could be derived. Such bounds are unique in their dependence on the bandwidth of the gain process.

In discrete time, moment stability for systems with finite correlation times in the gain process has been studied, and stability regions have been identified by directly calculating growth rates of the moment equations as a function of moment number and length of correlation time. Our results do not support the conjecture that one could stabilize an unstable feedback system by the use of multiplicative noise in the feedback path, although some unexpected results are obtained in the case where the correlation function becomes negative.

1.5. Failure Detection Methods

Professors A.S. Willsky and N.R. Sandell, Jr. and Dr. K.P. Dunn have initiated studies into the development of on-line failure detection and compensation and false measurement rejection methods. The original direction of this work has come from NASA/Langley, and we are aiming our work at the design of a system that can be used in digital fly-by-wire control systems.

The approach that has been chosen is a generalized likelihood ratio (GLR) technique [1] in which we make all detection decisions based on the residuals of a single Kalman filter.

Computer algorithms for the generation of the failure detection system gains for a wide variety of actuator and sensor failure modes have been developed. In addition methods for reducing the amount of on-line computer storage and computation required by the GLR system have been investigated and we have examined the effects of parameter variations on system performance.

Reference

1. A.S. Willsky and H.L. Jones, "A Generalized Likelihood Ratio Approach to State Estimation in Linear Systems Subject to Abrupt Changes", Proc. of 1974 IEEE Conference on Decision and Control; submitted to IEEE Trans. on Aut. Control.

1.6. Stochastic Control

The following problems in stochastic control have been investigated:

i) Control of finite state memory stochastic systems

ii) One-step delay LQG problem

iii) Stochastic Control of Linear Systems by the Multiple Time Scales Method

1.6.1. Control of Finite-State Finite-Memory Stochastic Systems

Professor N.R. Sandell, Jr. has worked on the finite-state finite memory stochastic control problem. The details of this work may be found in references [1] and [2].

For this problem, the usual technique of stochastic dynamic programming did not apply. Instead, optimality conditions have been obtained by deriving an equivalent deterministic optimal control problem.

A FSFM minimum principle has been obtained via the equivalent deterministic problem. The minimum principle has suggested the development of a numerical optimization algorithm, the min-H algorithm. The relationship between the sufficiency of the minimum principle (which has been, in general, only a necessary condition) and the informational properties of the problem has been investigated.

Dynamic programming functional equations for the FSFM problem have also been obtained from the equivalent deterministic problem. Both the finite and infinite horizon cases have been considered. Numerical solution of the functional equations has been discussed.

To illustrate the general theory, a problem of hypothesis testing with 1-bit memory has been investigated. The discussion has illustrated the application of control theoretic techniques to information processing problems.

This work has suggested that optimal decentralized controllers attempt to signal, or communicate with, each other. Since these signalling strategies introduce nonlinearities into even linear control systems, it is impossible at present to compute the optimal decentralized strategies. However, the nature of the optimal solution suggests the desireability of designing suboptimal linear controllers that are not completely isolated, but communicate across mutual interfaces.

Preliminary investigation indicates that the suboptimal controller design for fixed interface structure can be formulated as a parameter optimization problem. Development of appropriate software is underway. It is planned to test the resulting algorithms on a number of models, including a model of the F-8 aircraft at a high angle of attack where the coupling between the longitudinal and lateral dynamics becomes significant.

References

1. N.R. Sandell, Jr., Control of Finite-State, Finite-Memory Stochastic Systems, Ph.D. Thesis, M.I.T., 1974.

2. N.R. Sandell, Jr. and M. Athans, "A Finite-State, Finite-Memory Minimum Principle", Proceedings 1975 IEEE Conference on Decision and Control, Houston, Texas, pp. 637-644, December 1975 (also to appear Journal of Optimization Theory and Applications).

1.6.2. One-Step Delay LQG Problem

In the classical stochastic control case (single controller with access to all past system measurements) the linear-quadratic-Gaussian (LQG) problem provides a convenient framework for a wide class of practical problems. In the non-classical case, the solution to the LQG problem is unknown, although it is known the optimal control laws are in general nonlinear (Witsenhausen counter example). Research has been directed towards finding classes of problems for which the optimal control laws are linear, and at understanding cases in which the control laws are nonlinear. Results have been obtained for the one-step delay LQG problem and the control-sharing LQG problem. The solution to the former is linear, to the latter, nonlinear. The nonlinear solution clearly indicates that coding (a nonlinear operation) is required for communication between the controllers.

Reference

1. N.R. Sandell and M. Athans, "Solution of Some Non-Classical LQG Stochastic Decision Problems", to appear in IEEE Trans. on Auto. Control, Vol. AC-19, No. 2, April 1974.

1.6.3. Stochastic Control of Linear Systems by the Multiple Time Scales Method

Natural phenomena in engineering systems often evolve in widely separated time scales. Specifically, one can point to the separation of the phugoid and short period modes in the longitudinal dynamics of an aircraft; the separation of the rigid body and flexure modes of aircraft, spacecraft, and launch vehicles; and the separation of the Schuler and earth-rate frequencies in inertial navigation systems. Effects of this type are often exploited in other engineering sciences, but have only recently been introduced into control science by Kokotovic and a few other workers.

Mr. D. Teneketzis and Professor N.R. Sandell, Jr. have investigated the application of multiple time scale methods to the stochastic linear regulator problem. Using the methodology of singular perturbation theory, a hierarchically structured controller has been obtained that is optimal in the limit as the separation of the time scales become infinite. The higher levels of the controller deal with the slower system dynamics and do not require any information from the lower levels. The lower levels of the controller deal with the faster system dynamics and require information from higher levels. Thus the higher level portion of the control algorithm can be iterated at a much slower rate than would be possible by a single time scale design with an attendant saving in on-line computation. Significantly, this savings is not achieved at the expense of additional off-line computation. Indeed, the off-line computations of the multiple time-scale design are reduced in size and are better conditioned than those of the single time scale design.

The results have been documented in the M.S. thesis of Mr. Teneketzis.

Reference

1. D. Teneketzis, "Perturbation Methods in Decentralized Stochastic Control", ESL-R-664, June 1976 (also S.M. Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts).

1.7. Adaptive Control

Introduction

The need for adaptive control is necessitated by the need to take into account explicit variations in the parameters of the open loop system. Several adaptive techniques are available in the literature each requiring different amounts of real time computation.

What is not clear from an engineering viewpoint is when adaptive control should be used. If one follows the modern trend in evaluating the performance of an adaptive stochastic control algorithm not only by its identification accuracy, but rather by the performance improvement (as measured by a well defined performance index) of the overall system, then it is desirable to have an idea of

(a) the robustness of a non-adaptive, i.e. fixed gain, stochastic control algorithm

(b) a "worst-case" type of performance that might be expected if parameter identification cannot be carried out.

To properly answer these questions is the subject of a long range research program. Several research projects have been undertaken to answer these questions. We feel that partial results we have obtained provide a basis for understanding not only the issue of when one should use adaptive control, but hopefully shed some light upon the development of adaptive control algorithms which are computationally more efficient than present dual methods.

Of fundamental importance is the interaction of system uncertainty, as exhibited by large parameter variations in the open loop dynamics, <u>and</u> the trade-off parameters that appear in the performance index to be optimized.

Robustness of Linear Quadratic Designs

In section 2.5, Part 1 of the report, we have described results on robustness of LQ designs. The implications of these results on adaptive control have been investigated.

Probing and Learning in Adaptive Control

See section 2.7, Part 1 of the report.

Adaptive and Communications Aspects of Non-classical Control Theory

Mr. David Castagnon with Professors Athans and Sandell have been investigating adaptive and communication aspects of non-classical stochastic control theory. These problems are characterized by information patterns available to the controller which are not necessarily of the type occurring in stochastic control.

The problem considered was one where perfect recall is not present. This is a stochastic control problem whereby, through memory limitations or other factors, the controller may lose information from time to time. A mathematical framework,similar to Witsenhausen's was developed, using this framework, general Necessary Conditions were developed for the existence of a solution. Similarly, several results based on duality properties were established which produced some feasible techniques to obtain solutions to the problem.

Multiple Model Adaptive Control

Professor M. Athans, Dr. K-P Dunn, and Mr. B.F. Gong have studied several theoretical issues related to adaptive control of nonlinear stochastic systems with special emphasis on problems associated with the design of adaptive stability augmentation systems for aircraft. The equations of the F-8 Digital-Fly-By-Wire aircraft have been used as the main vehicle for understanding the basic issues.

Particular attention has been given to the so-called Multiple Model Adaptive Control (MMAC) method which blends hypothesis testing ideas with Linear Quadratic Gaussian design techniques to design a nonlinear adaptive control system. The MMAC method requires that the linearized stochastic dynamics of the nonlinear system (e.g., the linearized equations of the aircraft) are available at several operating conditions (in the case of aircraft corresponding to equilibrium flight at different altitude and Mach numbers).

There are two basic tradeoffs that have to be considered.

1) From the point of view of control system performance, the larger the number of linear models available, the better the control system.

2) From the point of view of computation, the greater the number of models, the larger the

(a) core memory requirements

(b) real-time computations.

What we have been developing is a systematic approach, employing dynamic programming, to arrive at a scientific way of selecting the number of models in the MMAC approach taking into account the tradeoffs of performance vs. computational complexity. The results of this work have been documented in the S.M. thesis of B.F. Gong.

Reference

1. B.F. Gong, "On Adaptive Stochastic Control", M.S. Thesis, May 1976, Massachusetts Institute of Technology, Cambridge, Massachusetts.

1.8. Infinite Dimensional Linear Systems

Over the past five years Professor Mitter has worked on problems of estimation and control for a large class of linear infinite dimensional systems. This class includes systems with time-delay, systems described by parabolic equations and systems described by second order hyperbolic equation. A theory which is formally analogous to the linear-quadratic-Gaussian theory for finite dimensional linear systems is now available. Details of this can be found in the forthcoming monograph [1].

Reference

1. A. Bensoussan, M.C. Delfour, and S.K. Mitter, Representation and Control of Linear Infinite Dimensional Systems, to appear.

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1. Athans, M. "System Aspects of Air Traffic Control". Proceedings 4th Hawaii International Conference on Systems Sciences. Honolulu, Hawaii. January 1971.

2. Athans, M. "Optimal Measurement Strategies for Linear Stochastic Systems". Technical Note 1971-12, MIT Lincoln Laboratory, Lexington, Ma. March 1971.

3. Athans, M. "On Optimal Allocation and Guidance Laws for Linear Interception and Rendezvous Problems". IEEE Transactions on Aerospace Electronic Systems. Volume AES-7. Number 5. pp. 842-853. September 1971. (Also, Proceedings 1970 Joint Automatic Control Conference, Atlanta, Georgia, June 1970.)

4. Athans, M. "On the Design of P-I-D Controllers Using Optimal Regulator Theory". Automatica. Volume 7. pp. 643-647. September 1971. (Also, ESL-P-432.)

5. Athans, M. "The Compensated Kalman Filter". Proceedings Symposium on Nonlinear Estimation. University of California. San Diego, California. September 1971.

6. Athans, M. "The Role and Use of the Linear Quadratic Gaussian Problem in Control System Design". IEEE Transactions on Automatic Control. Volume AC-16. Number 6, pp. 529-552. December 1971. (Also, ESL-P-438.)

7. Athans, M. "On the Determination of Optimal Costly Measurement Strategies for Linear Stochastic Systems". Proceedings 5th IFAC Congress. Paris, France. June 1972.

8. Athans, M. "An Example for Understanding Nonlinear Prediction Algorithms". ESL-TM-553. June 1974.

9. Athans, M. "Application of Modern Control Theory to Scheduling and Path-Stretching Maneuvers of Aircraft in the Near Terminal Area". ESL-R-574. October 1974.

10. Athans, M. "Theoretical and Practical Problems in the Design of Stochastic Large Scale Systems". Iranian Journal Science and Technology. Volume 3. Number 4. January 1975.

11. Athans, M. "Survey of Decentralized Control Methods". Annals of Economic and Social Measurement. Volume 4. Number 2. pp. 345-355. April 1975. (Also, ESL-P-555.)

12. Athans, M. "Trends in Modern System Theory". ESL-P-647. Keynote address delivered at the Engineering Foundation Conference on Chemical Process Control, Asilomar Conference Grounds, California. January 1976.

13. Athans, M., et al. "The Stochastic Control of the F8-C Aircraft Using the Multiple Model Adaptive Control (MMAC) Method". Proceedings 1975 IEEE Conference on Decision and Control. Houston, Texas. pp. 217-228. December 1975. (Also, ESL-P-622.)

14. Athans, M. and Porter, L.W. "An Approach to Semiautomated Optimal Scheduling and Holding Strategies for Air Traffic Control". Proceedings 1971 Joint Automatic Control Conference. St. Louis, Missouri. pp. 536-545. August 1971. (Also, ESL-P-437.)

15. Athans, M., Whiting, R.H., and Gruber, M. "A Suboptimal Estimation Algorithm with Probabilistic Editing for False Measurements with Applications to Target Tracking with Wake Phenomena". Proceedings 6th Symposium on Nonlinear Estimation. San Diego, California. September 1975. (Also, accepted IEEE Transactions on Automatic Control.)

16. Athans, M. and Willner, D. "A Practical Scheme for Adaptive Aircraft Flight Control Systems". Proceedings Symposium on Parameter Estimation Techniques and Applications in Aircraft Flight Testing. NASA Flight Research Center, Edwards AFB, California. April 1973.

17. Athay, T.M. and Sandell, N.R. "An Iterative Decoupling Solution Method for Large Scale Lyapunov Equations". Proceedings 8th Annual Southeastern Symposium on System Theory. University of Tennessee, Knoxville, Tennessee. April 26-27, 1976.

18. Bensoussan, A., Delfour, M.C., and Mitter, S.K. "Optimal Filtering for Linear Stochastic Hereditary Differential Systems". Presented at the Decision and Control Conference. New Orleans, Louisiana. December 1972.

19. Bensoussan, A., Delfour, M.C., and Mitter, S.K. "Representation and Qualitative Properties of Infinite Dimensional Linear Systems". Part I. ESL-P-602. June 1975. (This report forms a chapter of a forthcoming monograph entitled: "Representation and Control of Infinite Dimensional Linear Systems".)

20. Bensoussan, A., Delfour, M.C., and Mitter, S.K. "Optimal Control of Linear Systems with a Quadratic Cost Criterion". Part II. ESL-P-603. June 1975. (This report forms a chapter of a forthcoming monograph entitled: "Representation and Control of Infinite Dimensional Linear Systems".)

21. Bensoussan, A., Delfour, M.C., and Mitter, S.K. "Applications". Part III. ESL-P-604. June 1975. (This report forms a chapter of a forthcoming monograph entitled: "Representation and Control of Infinite Dimensional Linear Systems".)

22. Bensoussan, A., Delfour, M.C., and Mitter, S.K. "Representation Theory for Linear Infinite Dimensional Continuous Time Systems". ESL-P-640. December 1975. (To appear in Proceedings of the Conference on Algebraic System Theory. Udine, Italy, June 1975, Springer-Verlag in Mathematics Lecture Notes Series.)

23. Bertsekas, D.P. "Control of Uncertain Systems with a Set Membership Description of the Uncertainty". Ph.D. Dissertation. June 1971.

24. Bertsekas, D.P. "On the Separation Theorem for Linear Systems, Quadratic Criteria, and Correlated Noise". Submitted to IEEE Transactions on Automatic Control.

25. Bertsekas, D.P. and Mitter, S.K. "Steepest Descent for Optimization Problems with Non-differentiable Cost Functionals". Proceedings Fifth Annual Princeton Conference on Information Sciences and Systems. Princeton, New Jersey. March 1971.

26. Bertsekas, D.P. and Mitter, S.K. "A Descent Numerical Method for Optimization Problems with Nondifferentiable Cost Functionals". SIAM Journal on Control. Volume 11. Number 4. pp. 637-652. November 1973.

27. Bertsekas, D.P. and Rhodes, I.B. "On the Minimax Reachability of Target Sets and Target Tubes". Automatica. Volume 7. Number 2. March 1971.

28. Bertsekas, D.P. and Rhodes, I.B. "Recursive State Estimation for a Set Membership Description of Uncertainty". IEEE Transactions on Automatic Control. Volume AC-16. Number 2. April 1971.

29. Blanvillain, P.J. and Johnson, T.L. "Specific-Optimal Control with a Dual Minimal-Order Observer-based Compensator". Submitted to International Journal of Control, May 1976.

30. Carpenter, Jr., C.H. "Some Control and Communication Aspects of Stochastic Systems". ESL-R-481. May 1972. (Also, M.S. Thesis.)

31. Castanon, D. and Athans, M. "On Stochastic Dynamic Stackelberg Strategies". Proceedings 1975 IFAC Congress. Cambridge, Ma. August 1975. (Also, ESL-P-561 and Automatica. Volume 12. Number 2. pp. 177-183. March 1976.)

32. Chang, C.B., Whiting, R.H. and Athans, M. "On the State and Parameter Estimation for Maneuvering Re-entry Vehicles". Proceedings 6th Symposium on Nonlinear Estimation. San Diego, California. September 1975. (Also, accepted in IEEE Transactions on Automatic Control.)

33. Chong, C-Y. "On the Decentralized Control of Large-Scale Systems". ESL-R-503. June 1973. (Also, Ph.D. Thesis.)

34. Chong, C-Y., and Athans, M. "On the Stochastic Control of Linear System with Different Information Sets". IEEE Transactions on Automatic Control. Volume AC-16. pp. 423-430. October 1971. (Also, Proceedings 1971 Joint Automatic Control Conference, pp. 102-110 and ESL-P-439.)

35. Chong, C-Y. and Athans, M. "Hierarchical Decomposition for a Stochastic Optimization Problem". Proceedings 1973 Princeton Conference. Princeton, New Jersey. March 1973.

36. Chong, C-Y. and Athans, M. "On the Periodic Coordination of Linear Stochastic Systems". Automatica. July 1976. (Also, Proceedings 1975 IFAC Congress, August 1975 and ESL-P-570.)

37. Delfour, M.C., McCalla, C., and Mitter, S.K. "Stability and the Infinite-time Quadratic Cost Problem for Linear Hereditary Differential Systems". SIAM Journal on Control. Volume 13. Number 1. January 1975.

38. Delfour, M.C. and Mitter, S.K. "Le Contrôle Optimal de Systèmes Gouvernés par des Equations Différentielles Héréditaires Affines". C.R. Adad. Sc. Paris. 272. pp. 1715-1718. 1971.

39. Delfour, M.C. and Mitter, S.K. "Systèmes d'équations Différentielles Héréditaires à Retards Fixed. Théorèmes d'existence et d'unicité". C.R. Acad. Sc. Paris. 272. pp. 382-385. 1971.

40. Delfour, M.C. and Mitter, S.K. "Systèmes d'équations Différentielles Héréditaires à Retards Fixes. Une Classe de Systèmes Affines et le Problème du Système Adjoint". C.R. Acad. Sc. Paris. 272. pp. 1109-1112. 1971.

41. Delfour, M.C. and Mitter, S.K. "Controllability and Observability of Infinite-Dimensional Systems". SIAM Journal on Control. Volume 10, Number 2. May 1972.

42. Delfour, M.C. and Mitter, S.K. "Controllability, Observability and Optimal Feedback Control of Affine Hereditary Differential Systems". SIAM Journal on Control. Volume 10. Number 2. May 1972.

43. Delfour, M.C. and Mitter, S.K. "Control of Hereditary Differential Systems". Proceedings 1972 IFAC Congress. Paris, France. June 1972.

44. Delfour, M.C. and Mitter, S.K. "Hereditary Differential Systems with Constant Delays I: General Case". Journal of Differential Equations. Volume 12, Number 2. pp. 213-235. September 1972.

45. Delfour, M.C. and Mitter, S.K. "Control of Affine Systems with Memory". Presented at IFIPS Conference on Optimization Techniques, Rome, Italy. May 1973. (Also, appeared in Springer-Verlag in their Lecture Notes Series.)

46. Delfour, M.C. and Mitter, S.K. "L^2-Stability, Stabilizability Infinite Time Quadratic Cost Control Problem and Operator Riccati Equation for Linear Autonomous Hereditary Differential Systems". SIAM Journal on Control. Volume 13. Number 1. January 1975.

47. Delfour, M.C. and Mitter, S.K. "Hereditary Differential Systems with Constant Delays II: A Class of Affine Systems and the Adjoint Problems". Journal of Differential Equations. Volume 18. Number 1. pp. 18-28. May 1975.

48. Dunn, K.P. and Rhodes, I.B. "Cone-Bounded Nonlinearities and Mean-Square Bounds--Smoothing and Prediction Bounds". Automatica. Volume 11. pp. 517-523. 1975. (Also, Proceedings IFAC Conference 1975, Boston.)

49. Eckberg, A.E. "Algebraic Systems Theory with Applications to Decentralized Control". Ph.D. Dissertation. June 1973.

50. Gaidos, J.I. "Information and Distortion in Filtering Theory". Ph.D. Dissertation. January 1975.

51. Geering, H.P. "Optimal Control Theory for Non-Scalar Valued Performance Criteria". Proceedings of Fifth Annual Princeton Conference on Information Sciences and Systems. Princeton, New Jersey. March 1971. (Also, Ph.D. Thesis and ESL-R-445.)

52. Geering, H.P. and Athans, M. "Optimal Control Theory for Non-Scalar Valued Criteria". Proceedings Fifth Annual Princeton Conference on Information Sciences and Systems. Princeton, New Jersey. pp. 479-483. March 1971.

53. Geering, H.P. and Athans, M. "Necessary and Sufficient Conditions for Differentiable Non-Scalar-Valued Functions to Attain Extrema". IEEE Transactions on Automatic Control. Volume AC-18. pp. 129-132. 1973.

54. Geering, H.P. and Athans, M. "The Infimum Principle". IEEE Transactions on Automatic Control. Volume AC-19. Number 5. pp. 485-494. October 1974. (Also, 1974 Decision and Control Conference and ESL-P-492.)

55. Gong, B.F. "On Adaptive Stochastic Control". S.M. Thesis. May 1976.

56. Hager, W.W. and Mitter, S.K. "Lagrange Duality Theory for Convex Control Problems". To appear in SIAM Journal on Control and Optimization. Volume 14. Number 5. August 1976. (Also, ESL-P-630.)

57. Johnson, T.L. "Finite-State Compensators for Physical Systems". ESL-TM-658. April 1976.

58. Johnson, T.L., Athans, M., and Skelton, G.B. "Optimal Control-Surface Locations for Flexible Aircraft". IEEE Transactions on Automatic Control. Volume AC-16. Number 4. pp. 320-331. August 1971.

59. Kam, P.Y. and Willsky, A.S. "Some Estimation Problems Involving Space-Time Stochastic Processes". 1976 IEEE Conference on Decision and Control. Clearwater, Florida. Invited paper. December 1976.

60. Kramer, L.C. "On Stochastic Control and Optimal Measurement Strategies". ESL-R-463. November 1971. (Also, Ph.D. Thesis.)

61. Kramer, L.C. and Athans, M. "On Linear Stochastic Systems with Coupled Measurement and Dynamic Control". Proceedings 1972 Princeton Conference on Circuits and Systems. March 1972. (Also, Ricerche di Automatica, pp. 144-157, 1972.)

62. Kramer, L.C. and Athans, M. "On Simultaneous Choice of Dynamic Control and Measurement Strategies for Stochastic Systems". Proceedings of 1972 JACC. pp. 176-182. August 1972.

63. Kramer, L.C. and Athans, M. "On the Application of Deterministic Optimization Methods to Stochastic Control Problems". IEEE Transactions on Automatic Control. Volume AC-19. Number 1. February 1974. (Also, Proceedings of 1972 JACC, pp. 183-190, August 1972.)

64. Ku, R. "Adaptive Control of Stochastic Linear Systems with Unknown Parameters". ESL-R-477. May 1972.

65. Ku, R. and Athans, M. "Open-Loop-Feedback-Optimal Adaptive Stochastic Control of Linear Systems". Proceedings 3rd IFAC Symposium on Ident. and System Parameter Estimation. Netherlands. June 1973.

66. Ku, R. and Athans, M. "On the Adaptive Control of Linear Systems Using the Open-Loop-Feedback-Optimal Approach". IEEE Transactions on Automatic Control. Volume AC-18. Number 5. pp. 489-493. October 1973. (Also, Proceedings 1972 IEEE Decision and Control Conference, New Orleans, La., December.)

67. Kwong, R. "On a Stochastic Control Method for Weakly Coupled Linear Systems". ESL-R-480. May 1972.

68. Kwong, R., Chong, C-Y., and Athans, M. "On Stochastic Control Systems Design Methods for Weakly Coupled Large Scale Linear Systems". Proceedings 1972 Joint Automatic Control Conference. pp. 220-229.

69. Levine, W.S., Johnson, T.L., and Athans, M. "Optimal Limited State Variable Feedback Controllers for Linear Systems". IEEE Transactions on Automatic Control. (Special Issue) Volume AC-16. Number 6. pp. 785-793. December 1971.

70. Lo, J. T-H. and Willsky, A.S. "Estimation for Rotational Processes with One Degree of Freedom I: Introduction and Continuous Time Processes". IEEE Transactions on Automatic Control. Volume AC-20. Number 1. pp. 10-21. February 1975. (Also, ESL-P-506.)

71. Marcus, S.I. "Estimation and Analysis of Nonlinear Stochastic Systems". ESL-R-601. June 1975. (Also, Ph.D. Dissertation.)

72. Marcus, S.I. and Willsky, A.S. "The Use of Harmonic Analysis in Suboptimal Estimator Design". 1976 IEEE Conference on Decision and Control. Clearwater, Florida. December 1976.

73. Marcus, S.I. and Willsky, A.S. "Algebraic Structure and Finite Dimensional Nonlinear Estimation". Mathematical Systems Theory. Edited by G. Marchesini and S.K. Mitter. Springer-Verlag. 1976. (Also, submitted to SIAM Journal Mathematical Analysis.)

74. Martin, D.N. and Johnson, T.L. "Stability Criteria for Discrete-Time Systems with Colored Multiplicative Noise". Proceedings of the 1975 IEEE Conference on Decision and Control. Houston, Texas.

75. Martin, D.N. and Johnson, T.L. "Stability Criteria for Continuous-Time Systems with Colored Multiplicative Noise". Proceedings of the 1976 Conference on Decision and Control. Clearwater, Florida. December 1976. (Also, to appear in IEEE Transactions on Automatic Control.)

76. Mitter, S.K. "The Infinite Time Quadratic Cost Problem for a Class of Infinite Dimensional Systems". Delay and Functional Differential Equations and Their Applications. Academic Press, Inc. 1972. (Also, Presented at the Park City Conference on Functional Differential Equations. Park City, Utah, March 1972.)

77. Mitter, S.K. and Foulkes, R. "Controllability and Pole Assignment for Discrete Time Linear Systems Defined Over Arbitrary Fields". SIAM Journal on Control. Volume 9. Number 1. February 1971.

78. Mitter, S.K. and Vinter, R. "Filtering for Linear Stochastic Hereditary Differential Systems, in Control Theory, Numerical Methods and Computer Systems Modelling". Edited by A. Bensoussan and J.L. Lions. Springer-Verlag, New York. pp. 1-21. 1975. (Also, presented at the International Symposium on Control Theory, Numerical Methods and Computer System Modeling, Institut de Recherche D'Informatique et D'Automatique, France. June 1974.)

79. Platzman, L.K. and Johnson, T.L. "A LQG Control Problem with Innovations-Feedthrough Solution". IEEE Transactions on Automatic Control. October 1976.

80. Safonov, M. "On the Sensitivity of Linear Quadratic Feedback Systems--A Brief Survey". ESL-TM-649. 1976.

81. Safonov, M. and Athans, M. "Gain and Phase Margin for Multiloop LQG Regulators". 1976 IEEE Conference on Decision and Control. Clearwater, Florida. December 1976. (Also, ESL-P-654 and submitted to the IEEE Transactions on Automatic Control.)

82. Sandell, N.R. "Optimal Linear Tracking Systems". ESL-R-456. September 1971. (Also, S.M. Thesis.)

83. Sandell, N.R. "Control of Finite-State, Finite-Memory Stochastic Systems". ESL-R-545. June 1974. (Also, Ph.D. Thesis.)

84. Sandell, N.R. "On Newton's Method for Riccati Equation Solution". IEEE Transactions on Automatic Control. Volume AC-19. Number 3. pp. 254-255. June 1974.

85. Sandell, N.R. "On Open Loop and Closed Loop Nash Strategies". IEEE Transactions on Automatic Control. Volume AC-19. Number 4. August 1974.

86. Sandell, N.R. and Athans, M. "On 'Type-ℓ' Multivariable Linear Systems". Automatica. Volume 9. pp. 131-136. 1973. (Also, ESL-P-465.)

87. Sandell, N.R. and Athans, M. "Comments on a Solution for a Class of Stochastic Cooperative Games". International Journal on Control. Volume 20. Number 1. pp. 169-170. 1974.

88. Sandell, N.R. and Athans, M. "Control of Finite State Finite Memory Stochastic Systems". Proceedings 1974 Princeton Conference on Information Sciences and Systems. March 1974.

89. Sandell, N.R. and Athans, M. "Solution of Some Non-Classical LQG Stochastic Decision Problems". IEEE Transactions on Automatic Control. Volume AC-19. Number 2. pp. 108-116. April 1974. (Also, Proceedings 1974 Joint Automatic Control Conference, Austin, Texas. June 1974.

90. Sandell, N.R. and Athans, M. "A Finite-State, Finite-Memory Minimum Principle Proceedings 1975 IEEE Conference on Decision and Control. Houston, Texas. pp. 637-644. December 1975.

91. Sandell, N.R. and Athans, M. "Design of a Generalized P-I-D Controller by Optimal Linear Control Theory". ESL-P-467. December 1971.

92. Sarris, A.H. "On Optimal Scheduling and Air Traffic Control in the Near Terminal Areas". ESL-R-459. September 1971.

93. Sarris, A.H. and Athans, M. "An Algorithm for Terminal Air Traffic Control". ESL-P-466. December 1971.

94. Segall, A. "Stochastic Processes in Estimation Theory". IEEE Transactions on Information Theory. Volume IT-22. Number 3. pp. 275-286. May 1976.

95. Segall, A., Davis, M.H.A., and Kailath, T. "Nonlinear Filtering with Counting Observations". IEEE Transactions on Information Theory. Volume IT-21. pp. 53-58. March 1975.

96. Segall, A. and Kailath, T. "Orthogonal Functionals of Independent-Increment Processes". IEEE Transactions on Information Theory. Volume IT-22. Number 3. pp. 287-298. May 1976.

97. Teneketzis, D. "Perturbation Methods in Decentralized Stochastic Control". ESL-R-664. June 1976. (Also, S.M. Thesis.)

98. Teneketzis, D. and Sandell, N.R. "Perturbation Methods in Decentralized Stochastic Control". International Conference on Systems Science. University of Patras. Patras, Greece. August 1976.

99. Teneketzis, D. and Sandell, N.R. "Regulator Design for Stochastic Systems by a Multiple Time Scales Method". 1976 IEEE Conference on Decision and Control. Clearwater, Florida. December 1976.

100. Tse, E. and Athans, M. "The Status of Observer Theory for Linear Systems". Proceedings 1971 Joint Automatic Control Conference. St. Louis, Mo. pp. 17-18. August 1971.

101. Tse, E. and Athans, M. "Adaptive Stochastic Control for a Class of Linear Systems". IEEE Transactions on Automatic Control. Volume AC-17. February 1972.

102. Tse, E. and Athans, M. "Observer Theory for Continuous Time Linear Systems". Information and Control. Volume 22. Number 5. pp. 405-434. June 1973.

103. Vandevenne, H.F. "Qualitative Properties of a Class of Infinite Dimensional Systems". ESL-R-479. May 1972. (Also, Ph.D. Thesis.)

104. Varaiya, P. "N-Player Stochastic Differential Games". SIAM Journal on Control. 1976. (Also, ESL-P-578.)

105. Willems, J.C. and Mitter, S.K. "Controllability, Observability, Pole Allocation, and State Reconstruction". IEEE Transactions on Automatic Control. Volume AC-16. Number 6. pp. 582-595. December 1971.

106. Willsky, A.S. "Some Estimation Problems on Lie Groups". Geometric Methods in System Theory. Edited by D.Q. Mayne and R.W. Brockett. Reidel Publishing Company. Netherlands. 1973. (Also, ESL-P-504.)

107. Willsky, A.S. "Invertibility Conditions for a Class of Finite State Systems Evolving on Groups". Proceedings of the Eleventh Annual Allerton Conference on Circuit and System Theory. October 1973. (Also, ESL-P-519.)

108. Willsky, A.S. "Fourier Series and Estimation on the Circle with Applications to Synchronous Communication I: Analysis". IEEE Transactions on Information Theory. September 1974. (Also, ESL-P-510.)

109. Willsky, A.S. "Fourier Series and Estimation on the Circle with Applications to Synchronous Communication II: Implementation". IEEE Transactions on Information Theory. September 1974. (Also, ESL-P-511.)

110. Willsky, A.S. "Invertibility of Finite Group Homomorphic Sequential Systems". Information and Control. Volume 27. Number 2. pp. 126-147. February 1975. (Also, ESL-P-518.)

111. Willsky, A.S. "The Use of Group-Theoretic Concepts in Solving Problems in Estimation and Control". 1976 IEEE Conference on Decision and Control. Clearwater, Florida. Invited paper. December 1976.

112. Willsky, A.S. and Lo, J. T-H. "Estimation for Rotational Processes with One Degree of Freedom II: Discrete Time Processes". IEEE Transactions on Automatic Control. Volume AC-20. Number 1. pp. 22-30. February 1975. (Also, ESL-P-507.)

113. Willsky, A.S. and Lo, J. T-H. "Estimation for Rotational Processes with One Degree of Freedom III: Application and Implementation". IEEE Transactions on Automatic Control. Volume AC-20. Number 1. pp. 31-33. February 1975. (Also, ESL-P-506.)

114. Willsky, A.S. and Marcus, S.I. "Estimation for Certain Nonlinear Stochastic Systems". Seminaires IRIA. Rocquencourt, France. May 1975.

115. Willsky, A.S., Marcus, S.I., and Martin, D.N. "On the Stochastic Stability of Linear Systems Containing Colored Multiplicative Noise". IEEE Transactions on Automatic Control. 1975.

116. Wong, P. and Athans, M. "Closed-Loop Structural Stability for Linear-Quadratic Optimal Systems". Accepted IEEE Transactions on Automatic Control and IEEE Conference on Decision and Control. Clearwater, Florida. December 1976 (Also, ESL-P-641.)

117. Baram, Y., "Information, Consistent Estimation and Dynamic System Identification". ESL-R-718. M.I.T., Cambridge, Massachusetts. November 1976.

118. Bensoussan, A., Delfour, M.C. and Mitter, S.K., "The Linear Quadratic Optimal Control Problem for Infinite Dimensional Systems over an Infinite Horizon; Survey and Examples". Proceedings of the 1976 IEEE Conference on Decision and Control. 76CH1150-2CS. December 1-3, 1976. Clearwater, Florida. pp.746-751.

119. Djaferis, T.E. and Mitter, S.K., "Exact Solution to Lyapunov's Equation Using Algebraic Methods". Proceedings of the 1976 IEEE Conference on Decision and Control. 76CH1150-2CS. December 1-3, 1976. Clearwater, Florida. pp. 1194-1200.

120. Horowitz, L. and Mitter, S.K., "An Example of an Infinite Dimensional Filtering Problem: Filtering for Gyroscopic Noise". Proceedings of the 1976 IEEE Conference on Decision and Control. 76CH1150-2CS. December 1-3, 1976. Clearwater, Florida. pp. 764-773.

121. Kam, P.K. and Willsky, A.S., "Some Modeling and Estimation Problems Involving Space-Time Stochastic Processes". Proceedings of the 1976 IEEE Conference on Decision and Control. 76CH1150-2CS. December 1-3, 1976. Clearwater, Florida. pp. 117-123.

122. Marcus, S.I. and Willsky, A.S., "The Use of Spherical Harmonics in Suboptimal Estimator Design". Proceedings of the 1976 IEEE Conference on Decision and Control. 76CH1150-2CS. December 1-3, 1976. Clearwater, Florida. pp. 198-203.

123. Safonov, M.G. and Athans, M., "Gain and Phase Margin for Multiloop LQG Regulators". Proceedings of the 1976 IEEE Conference on Decision and Control. 76CH1150-2CS. December 1-3, 1976. Clearwater, Florida. pp. 361-368.

124. Wong, P.K. and Athans, M., "Closed-Loop Structural Stability for Linear-Quadratic Optimal Systems". Proceedings of the 1976 IEEE Conference on Decision and Control. 76CH1150-2CS. December 1-3, 1976. Clearwater, Florida. pp. 1231-1239.

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This final report describes the research carried out by members of the Decision and Control Sciences Group at the Electronic Systems Laboratory, M.I.T. during the time period February 1, 1972 to January 31, 1977 with support extended by the Air Force Office of Scientific Research under Grant AF-AFOSR 72-2273.

The principal investigators were Professor Michael Athans and Professor Sanjoy K. Mitter. The contract monitors were Lt. Colonel W.J. Rabe and

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Research was carried out on the following main topics:

1. Modelling and Identification of Stochastic Systems;
2. Structure of Non-linear Stochastic Systems;
3. Linear and Non-linear Filtering;
4. Failure Detection Methods;
5. Stochastic Control;
6. Adaptive Control; and
7. Control and Estimation Problems for Systems with Time Delay.

Technical details of the research may be found in the reports, theses, and papers cited in the references. A list of publications supported wholly or partially by this grant is included at the end of this report.